

**General Certificate of Education (A-level) June 2013** 

**Mathematics** 

**MD02** 

(Specification 6360)

**Decision 2** 

## **Final**

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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## **Key to mark scheme abbreviations**

| M           | mark is for method   |
|-------------|--|
| m or dM     | mark is dependent on one or more M marks and is for method         |
| A           | mark is dependent on M or m marks and is for accuracy              |
| В           | mark is independent of M or m marks and is for method and accuracy |
| E           | mark is for explanation  |
| √or ft or F | follow through from previous incorrect result                      |
| CAO         | correct answer only  |
| CSO         | correct solution only  |
| AWFW        | anything which falls within  |
| AWRT        | anything which rounds to   |
| ACF         | any correct form   |
| AG          | answer given   |
| SC          | special case   |
| OE          | or equivalent  |
| A2,1        | 2 or 1 (or 0) accuracy marks                                       |
| −x EE       | deduct x marks for each error                                      |
| NMS         | no method shown  |
| PI          | possibly implied   |
| SCA         | substantially correct approach                                     |
| c           | candidate  |
| sf          | significant figure(s)  |
| dp          | decimal place(s)   |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q          | Solution                                  | Marks          | Total        | Comments   |
|------------|---|----------------|--------------|--|
| 1(a)       |   | 14             | E            | $ \begin{array}{c c} H & J \\ \hline 18 \mid 30 & 30 \mid 42 \end{array} $     |
|            | 0 9 9 14<br>C<br>0 9                      |                | [24]<br>[26] | 26 30 30 42<br>L<br>30 42  |
|            |   | M1<br>A1       |              | Forward pass, correct at <i>D</i> , <i>E</i> , <i>F</i> , <i>G</i> All correct |
|            |   | M1<br>A1       | 4            | Backward pass, correct at <i>H</i> , <i>I</i> , <i>G</i> ft All correct        |
| (b)        | C D G I J only                            | B1             | 1            |  |
| (c)        | 6   | B1ft           | 1            | Their (latest – earliest – 4)  |
| (d)        | H delayed by 4 K delayed by 5 New time 51 | E1<br>B1<br>B1 | 3            | 51 scores 3/3  |
|            | Total                                     |                | 9            |  |
| 2(a)       | 19  | B1             | 1            |  |
| (b)        | E   | B1             | 1            |  |
| (c)        | C   | B1             | 1            |  |
| (d)        | x = 8<br>$y = 13$                         |                |              |  |
|            | z = 39                                    | B1 × 3         | 3            |  |
| (e)        | 76  | B1             | 1            |  |
| <b>(f)</b> | 83  | B1             | 1            |  |
|            | Total                                     |                | 8            |  |

| Q    | Solution  | Marks          | Total | Comments   |
|------|---|----------------|-------|--|
| 3(a) | Reduce columns  |                |       |  |
|      | $ \begin{pmatrix} 0 & 12 & 13 & 2 & 0 \\ 25 & 32 & 11 & 20 & 20 \\ 5 & 12 & 2 & 8 & 25 \end{pmatrix} $  | M1<br>A1       |       |  |
|      | $     \begin{bmatrix}       15 & 17 & 21 & 35 & 15 \\       0 & 0 & 0 & 0 & 7     \end{bmatrix} $   |                |       |  |
|      | Reduce rows $( 0 12 1 \beta 2                             $   |                |       |  |
|      | 14 21 0 9 9<br>3 10 0 6 23<br>0 2 6 20 0<br>0 0 0 7   |                |       | AG   |
|      | k = 9   | B1             | 3     |  |
| (b)  | 4 lines drawn on given table<br>Subtract/add 2  | B1<br>M1       |       | Condone one slip   |
|      | $ \begin{pmatrix} 0 & 10 & 13 & 0 & 0 \\ 14 & 19 & 0 & 7 & 9 \\ 3 & 8 & 0 & 4 & 23 \\ 0 & 0 & 6 & 18 & 0 \\ 2 & 0 & 2 & 0 & 9 \end{pmatrix} $ | A1             |       | Correct table with 4 lines shown   |
|      | Subtract/add 3  (0 10 16 0 0)  11 16 0 4 6  0 5 0 1 20  0 0 9 18 0  2 0 5 0 9   | m1<br>A1       | 5     | Condone one slip  All correct with no errors seen, including 5 lines drawn |
| (c)  | Match XA, WC<br>+ VD, YE, ZB<br>or VE, YB, ZD   | M1<br>A1<br>A1 | 3     | And no extras  |
| (d)  | 525   | B1             | 1     |  |
|      | Total   |                | 12    |  |

| Q               |       | Solut | tion |       | Marks | Total | Comments  |
|-----------------|-------|-------|------|-------|-------|-------|---|
| 4               | Stage | State | From | Value |       |       |   |
|                 | 1     | Н     | K    | 18    |       |       |   |
|                 |       | I     | K    | 15    | B1    |       | All correct                                       |
|                 |       | J     | K    | 12    |       |       |   |
|                 | 2     | Ε     | Н    | (17)  | M1    |       | 7 values at stage 2                               |
|                 |       |       | I    | 15    |       |       |   |
|                 |       | F     | Н    | (15)  | m1    |       | Choosing max at $E$ , $F$ , $G$ (PI), but must be |
|                 |       |       | I    | 14    |       |       | using maximin                                     |
|                 |       |       | J    | 12    |       |       |   |
|                 |       | G     | I    | (14)  | A1    |       | All comment at stage 2                            |
|                 |       |       | J    | 12    | Aı    |       | All correct at stage 2                            |
|                 | 3     | В     | E    | 11    | m1    |       | 7 values at stage 3, must have scored M2          |
|                 |       |       | F    | (13)  | 1111  |       | earlier   |
|                 |       | C     | E    | 12    |       |       | carner  |
|                 |       |       | F    | 13    | A1    |       | All correct at stage 3                            |
|                 |       |       | G    | (14)  |       |       | The correct at stage 5                            |
|                 |       | D     | F    | (15)  |       |       |   |
|                 |       |       | G    | 14    |       |       |   |
|                 | 4     | A     | В    | 12    |       |       |   |
|                 |       |       | C    | (14)  | A1    |       | All correct (whole table)                         |
|                 |       |       | D    | 13    | B1    |       | For 14 as final value indicated or stated         |
|                 |       |       |      |       |       |       |   |
| Route A C G I K |       |       |      |       |       | 9     | Or reverse  |
|                 |       |       |      |       |       |       |   |
|                 |       |       |      | Total |       | 9     |   |

| Q             | Solution  | Marks     | Total | Comments   |
|---------------|---|-----------|-------|--|
| 5(a)          | R min -4, -5, -2 plays C  | B1        |       | Either C or E stated                             |
|               | J max 4, 1, 3 plays E   | B1        | _     | Both C and E stated                              |
|               |   | E1        | 3     | and all values shown                             |
| (b)           | maximin $R = -2 \neq 1 = minimax J$   | E1        | 1     | Correct values must be stated                    |
| (c)           | (For Juliet,) col E dominates col D   | E1        | 1     |  |
| (d)(i)        | Signs changed as J gains = R losses<br>Gains written as rows                                  | E1<br>E1  | 2     |  |
| ( <b>ii</b> ) | Let J play E prob $p$<br>F $(1-p)$  |           |       |  |
|               | If R plays A, J wins $4p$ B $5p - 3(1-p)$ C $-p + 2(1-p)$ [gives $4p$ , $8p - 3$ , $2 - 3p$ ] | M1<br>A1  |       | 2 correct expressions seen<br>All correct        |
|               | 2<br>1<br>0<br>-1<br>-2<br>-3   | m1<br>A1  |       | Must have 3 lines  All correct with values shown |
|               | Max at $8p - 3 = 2 - 3p$  | m1        |       | Identifies correct max from their graph          |
|               | $p = \frac{5}{11}$  | A1        |       |  |
|               | (J plays)E prob $\frac{5}{11}$ , F prob $\frac{6}{11}$  | A1<br>CSO | 7     |  |
| (iii)         | Value of game = $\frac{7}{11}$  | В1        | 1     |  |
|               | Total   |           | 15    |  |
|               | Total   |           | 13    |  |

| Q      |                       |   |                             | Sol   | ution   |   |   |  | Marks    | Total | Comments  |
|--------|-----------------------|---|-----------------------------|---|---|---|---|--|----------|-------|---|
| 6(a)   | P<br>1<br>0<br>0<br>0 | $ \begin{array}{c c} x \\ \hline -4 \\ \hline 2 \\ 1 \\ 1 \end{array} $ | y<br>-3<br>1<br>2<br>1      | z<br>-1<br>1<br>1<br>2  | r s<br>0 0<br>1 0<br>0 1<br>0 0   | 0<br>0<br>0<br>0  |   | 7 <u>alue</u> 0 25 40 30                                 | B2,1,0   | 2     | All correct, 3 rows correct   |
| (b)    | 0                     | 0   | $-1$ $\frac{1}{2}$          | $\frac{1}{2}$   | $\frac{1}{2}$   | 0   | 0 | 50<br>25<br>2  | B1<br>M1 |       | Pivot, x-col: 12.5, 40, 30 seen and correct pivot chosen Row operations |
|        | 0                     | 0   | $\frac{3}{2}$ $\frac{1}{2}$ | $\frac{1}{2}$ $\frac{3}{2}$   | $ \begin{array}{c} 2 \\ -\frac{1}{2} \\ -\frac{1}{2} \end{array} $                | 0   | 0 | $\frac{55}{2}$ $\frac{35}{2}$                            | A1       | 3     | All correct   |
| (c)(i) | 1 0                   | 0   | 0                           | $\frac{4}{3}$   | $\frac{5}{3}$   | $\frac{2}{3}$   | 0 | $\frac{205}{3}$  | B1       |       | Pivot, y-col: their 25, 55/3, 35 seen and correct pivot chosen          |
|        | 0 0                   | 0   | 1 0                         | $\begin{array}{c} - \\ 3 \\ \frac{1}{3} \\ \frac{4}{3} \end{array}$ | $ \begin{array}{c} 3 \\ \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{array} $ | $ \begin{array}{c} -\overline{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{array} $ | 0 | $ \begin{array}{r}                                     $ | M1       | 3     | Row operations  All correct   |
| (ii)   |                       | $P = \frac{2}{}$  |                             |   |   |   |   |  | B1<br>B1 |       | Condone optimal, etc  Ft on x and y                                     |
|        |                       |   | $0, \ t = \frac{1}{2}$      |   |   |   |   |  | B1ft     | 3     | All 3 must be stated  |

| Q      |                       |                   |                 | Sol      | lution             |    |   |          | Marks | Total | Comments          |
|--------|-----------------------|-------------------|-----------------|----------|--------------------|----|---|----------|-------|-------|-------------------|
| 6      | Altern                | ative             | ;               |          |                    |    |   |          |       |       | Comments as above |
|        |                       |                   |                 |          |                    |    |   |          |       |       |                   |
| (a)    | P<br>1<br>0<br>0<br>0 | х                 | <u>y</u>        | Z        | r                  | S  | t | Value    |       |       |                   |
|        | 1                     | _4                | -3              | -1       | 0                  | 0  | 0 | 0        |       |       |                   |
|        | 0                     | (2)               | 1               | 1        | 1                  | 0  | 0 | 25       |       | (2)   |                   |
|        | 0                     | 1<br>1            | 2<br>1          | 1        | 0                  | 1  | 1 | 40<br>30 |       | (2)   |                   |
| (b)    | 0                     | 1                 | 1               | 2        | U                  | U  | 1 | 30       |       |       |                   |
| (0)    | 1                     | 0                 | _1              | 1        | 2                  | 0  | 0 | 50       |       |       |                   |
|        | 0                     | 2.                | 1               | 1        | 1                  | 0  | 0 | 25       |       |       |                   |
|        | ő                     | 0                 | (3)             | 1        | -1                 | 2  | 0 | 55       |       | (3)   |                   |
|        | 0                     | 0                 | 1               | 3        | 2<br>1<br>-1<br>-1 | 0  | 2 | 35       |       | (3)   |                   |
|        |                       | Ü                 | -               | 3        | •                  |    | _ | 33       |       |       |                   |
| (c)(i) |                       |                   |                 |          |                    |    |   |          |       |       |                   |
| (-)(-) | 3                     | 0                 | 0               | 4        | 5<br>4<br>-1<br>-2 | 2  | 0 | 205      |       |       |                   |
|        | 0                     | 6                 | 0               | 2        | 4                  | -2 | 0 | 20       |       |       |                   |
|        | 0                     | 0                 | 3               | 1        | -1                 | 2  | 0 | 55       |       | (2)   |                   |
|        | 0                     | 0                 | 0               | 8        | -2                 | -2 | 6 | 50       |       | (3)   |                   |
|        |                       |                   |                 |          |                    |    |   |          |       |       |                   |
| (**)   | n 2                   | 205               |                 |          |                    |    |   |          |       |       |                   |
| (ii)   | $P = \frac{2}{3}$     | 3                 |                 |          |                    |    |   |          |       |       |                   |
|        | 1                     | 0                 | 55              |          |                    |    |   |          |       |       |                   |
|        | $x = \frac{1}{3}$     | $\frac{3}{2}$ , y | $=\frac{3}{3}$  | , z =    | = 0                |    |   |          |       |       |                   |
|        |                       |                   |                 |          |                    |    |   |          |       |       |                   |
|        | r = s                 | = 0,              | $t=\frac{2}{3}$ | <i>-</i> |                    |    |   |          |       | (3)   |                   |
|        |                       |                   | 3               | <u> </u> |                    |    |   | Total    |       | 11    |                   |
|        |                       |                   |                 |          |                    |    |   | Total    |       | 11    |                   |

| Q    | Solution  | Marks                             | Total                  | Comments   |
|------|---|-----------------------------------|------------------------|--|
| 7(a) | $S \stackrel{56}{\overbrace{\hspace{1cm}}} R_1$ $T_1 \stackrel{36}{\overbrace{\hspace{1cm}}} T$ $T_2 \stackrel{26}{\overbrace{\hspace{1cm}}} T$ | B1<br>B1                          | 2                      | Edges with values $\geq 56$ , 52<br>Edges with values $\geq 36$ , 26, 28       |
| b(i) | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | A 1 1 14 18 18 18 6.8 6.8 12 20 C | 0 <sup>4</sup> D<br>42 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$                          |
|      |   | M1                                |                        | initial diagram with forward/back flows  |
|      |   | A1                                |                        | Fully correct diagram  |
|      | $SR_1 A D T_1 T = 4$<br>$SR_1 B D T_1 T = 2$<br>$SR_2 C E T_3 T = 6$<br>$SR_2 B E T_2 T = 4$  | M1<br>A1                          | 5                      | One correct path and flow At least one other correct path and flow all correct |
| (ii) | $S R_2 B E T_3 T = 4$ Max flow 90   | B1                                |                        | (ignore connections to $S$ and $T$ )   |
|      | $R_1$ $18$ $0$ $18$ $0$ $18$ $0$ $0$ $0$ $0$ $0$ $0$ $0$ $0$ $0$ $0$  | B1                                | 2                      |  |
| (c)  | Cut through (shown)   | B1                                |                        | PI by correct list   |
|      | $AT_{1}, DT_{1}, DT_{2}, ET_{2}, ET_{3}, CT_{3}$  | B1                                | 2                      | OE   |
|      | Total   |                                   | 11                     |  |
|      | TOTAL   |                                   | 75                     |  |